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Comments on the Colliding Beam Fusion Reactor Proposed by Rostoker, Binderbauer and Monkhorst for Use with the p-^{11}B Fusion Reaction

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We examine the various dissipative processes affecting the "colliding beam fusion reactor" (CBFR) recently proposed by Rostoker, Binderbauer and Monkhorst (RBM) for use with the p-^{11}B fusion reaction. We conclude that the CBFR equilibrium cannot be sustained for long enough to permit net fusion gain, because of the many collisional processes which occur orders of magnitude faster than fusion, and result in particle loss, energy dissipation, and/or detuning of the resonant energy for the p-^{11}B reaction. Fokker-Planck analyses of each process are performed. The calculations of RBM are critically reviewed to elucidate the source of disagreements with the present work. We also briefly discuss technology issues, especially the inefficiency of beam trapping in the device, and the unavailability of compact beam sources as assumed by RBM for naval applications.

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Comments on the Colliding Beam Fusion Reactor Proposed by Rostoker, Binderbauer and Monkhorst for Use with the p-^{11}B Fusion Reaction

1. Introduction

In a number of recent publications[1,2] and progress reports,[3] Rostoker, Binderbauer and Monkhorst (RBM) have advocated a colliding beam fusion reactor (CBFR) for aneutronic fusion based on the p-^{11}B reaction. This scheme was originally proposed as an energy source for all-electric naval ships, and the Office of Naval Research has supported concept studies at low levels for two years. In support of the ONR effort, NRL initiated a small effort by us to evaluate the p-^{11}B CBFR. This report summarizes our conclusions, which unfortunately are that the concept is fundamentally flawed, and cannot produce net fusion power.

Since the beginning of the fusion program, researchers have recognized the tremendous advantages of using the p-^{11}B fusion reaction. Since all of the fusion products are charged α-particles, it is widely (but not universally[4]) believed that a p-^{11}B reactor would be free of the radiation problems and bulky shielding associated with the production of fast neutrons, and fusion output energy could be converted directly and efficiently into electricity. The fuel would be cheap and abundant. However, there are formidable obstacles to the use of this reaction. As shown in Fig. 1, the reactivity is large only if the relative energy between a colliding proton and ^{11}B nucleus is very high, in the vicinity of $E_0 = 600$ keV, where there is a resonant peak σ_{F0} in the fusion cross-section $\sigma_F(E)$. If E is off resonance by as little as ±150 keV, $\sigma_F(E)$ falls to about half of σ_{F0}. If the protons and boron are thermally distributed at a temperature of 650 keV, the reaction rate is only about a third of what it is if every p-^{11}B encounter occurs at exactly energy E_0. Since the fuel must be so energetic, and the energy output is only 8.7 MeV per fusion reaction, the output fusion power (before conversion to electricity) can be no more than about 15 times the power invested in injected protons, even if proton acceleration is 100% efficient and every injected proton undergoes fusion. In addition, bremsstrahlung energy losses are a severe problem, because of the presence of nuclei with Z=5. When all of these factors, and others, are taken into account, the generally accepted conclusion[5] (as restated by RBM[1]) is that:

> "For a thermal p-^{11}B reactor, the electromagnetic radiation energy is greater than the nuclear energy produced, and a reactor that produces net energy is possible only if the conversion efficiency is nearly 100%."

In order to improve the energetics to the point where net energy gain might be possible, the primary objective of the RBM plan is to have nearly every encounter

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between a proton and a ^{11}B nucleus occur at relative energy close to E_0.[1-3,6-7] To bring this about, they plan to inject, confine, and maintain the protons and ^{11}B as separate cold beams:[6]

> "In order to exploit resonance in a steady-state reactor, it is necessary to maintain the proton and boron beams at an average energy difference of 580 keV. In addition, the temperatures of the beams must be substantially less than 140 keV."

Specifically, RBM propose to confine the reactants within a field-reversed configuration, as shown in Fig. 2, with the protons maintained as a rapidly rotating cold beam with a thin annular spatial profile. The ^{11}B ions form a fully stripped cold species, which may be a stationary component or a slowly rotating beam. The relative energy of the p and ^{11}B is E_0, but the temperatures of each species must be no more than tens of keV.[6] The electrons form a third species, which rotates at a speed intermediate between the p and ^{11}B beams. Because of the electrical current which is carried primarily by the rotating protons, the magnetic field reverses in crossing the ion ring. In various design options, RBM choose the magnetic swing to be in the regime of 100 to 200 kG, which implies that the product of the proton density n_p and the thickness Δr of the ring is $n_p\Delta r \approx \text{few} \times 10^{15}$ cm^{-2}. The radius of the proton ring is taken to be anywhere from 30 cm to 80 cm, and the thickness of the ring is assumed to be a few cm. Since the equilibrium is expected to be at least approximately of rigid rotor type, i.e. the ion rotational velocity is proportional to the radius r, it is important that the proton ring be thin to exploit the resonance in σ_F.

Forty years' experience provides convincing proof that fusion is a much more challenging problem than was envisioned by the pioneers. Many plasma confinement schemes have been studied. In considering a new confinement scheme, the standard procedure is to consider first the "classical" confinement and transport properties, i.e. those resulting from ordinary Coulomb collisions between the charged particles. These are relatively straightforward to understand, although detailed calculations may be difficult in complex confinement geometries. Normally, the only confinement schemes which are considered for further theoretical and experimental study are those for which the classical properties are found to be satisfactory. Typically, these more detailed investigations uncover a variety of instabilities and collective effects which may interfere with the achievement of adequate particle and/or energy confinement, and which must be overcome. However, the p-^{11}B CBFR proposed by RBM fails to pass the first test. Simple classical collisional properties and basic physical principles indicate that the equilibrium required for the CBFR cannot be sustained for long enough to provide fusion

gain. Various aspects of these problems have been elucidated in comments sent to *Science* magazine by seven scientists in response to Ref. 1.[8–13] All of these authors note that the p-^{11}B fusion reaction is at best marginal as regards energy gain, and that everything must break just right if the process is to have any chance. References 10–13 also present a variety of collisionality arguments similar to those presented here.

The organization of the paper is as follows. In Sec. 2, we summarize our notation. In Sec. 3, we discuss in some detail a variety of collision processes which destroy the CBFR equilibrium, on time scales much faster than fusion. In Sec. 4, we present an explicit calculation of the effect of p-^{11}B collisions on the proton temperature. This has been an ongoing point of contention between RBM and ourselves, and we also point out an error in the RBM analysis which led them to conclude that the protons can be cooled by p-^{11}B collisions. In Sec. 5, we briefly discuss the state of the art in beam source technology; indications are that the beam source for a p-^{11}B CBFR reactor would be too large to meet the Navy's need for shipboard power. We also offer brief comments on instability concerns, and on the limited experimental data base for field-reversing ion rings. In Sec. 6 we summarize our conclusions. Detailed Fokker-Planck calculations, which are the basis for Secs. 3 and 4, are deferred to the Appendix.

In drafting this paper, our intention has been to make the arguments accessible to the physics community at large, and not only to those who have a detailed background in plasma physics.

2. Summary of Notations

To facilitate comparisons between our work and the published papers of RBM, we have tried to use similar notation; however, in some cases we have chosen notations that are a bit clearer or more intuitive to us. The definition of quantities such as $\mathbf{V}_j$, $\mathbf{v}_j$ and T_j, is identical to the usage of RBM. We use cgs units.

Species: Subscripts p, B and e represent protons, ^{11}B nuclei and electrons, respectively.

Macroscopic parameters and distribution functions:

- n_j is the number density of species j at a particular spatial location; j = p, B, or e.
- $\rho_j = m_j n_j$ is the mass density of species j at a particular spatial location.
- $\mathbf{V}_j$ is the mean or fluid velocity of species j at a particular spatial location, defined as

$$\mathbf{V}_j = \int d^3\mathbf{v}\,\mathbf{v}\,f_j(\mathbf{v})\,. \tag{1}$$

- $f_j(\mathbf{v})$ is the velocity distribution function of species j, normalized to unity.
- V_j is the magnitude of the vector velocity $\mathbf{V}_j$.
- T_j is the temperature of species j, defined as

$$T_j = \int d^3\mathbf{v} \tfrac{1}{3} m_j \left(\mathbf{v} - \mathbf{V}_j\right)^2 f_j(\mathbf{v}). \tag{2}$$

  The definition of temperature does not assume that the distribution $f_j(\mathbf{v})$ is Maxwellian, or even that it is isotropic, but in thermal equilibrium it reduces to the usual definition.
- $v_j \equiv \sqrt{T_j/m_j}$ is the thermal velocity of species j.
- Δr is the width of the annular proton beam.

Cross-sections, rates, and characteristic times:

- $\sigma_F(E)$ is the p-^{11}B fusion cross-section, as a function of E, the relative energy between a colliding proton and ^{11}B nucleus. $\sigma_{F0} \equiv \sigma_F(E_0)$ is the peak value of $\sigma_F(E)$. RBM[15] cite two different references for $\sigma_F(E)$. Feldbacher and Heindler (1988)[16] give $\sigma_{F0} = 0.8\times10^{-24}$ at $E_0 = 620$ keV, while Becker, Rolfs and Trautvetter (1987)[17] give $\sigma_{F0} = 1.15\times10^{-24}$ at $E_0 = 580$ keV. In addition, RBM argue[18] that it may be possible to increase $\sigma_F(E)$ by a factor of 1.6 by spin-polarizing the fuel nuclei. Throughout this paper, we shall use the most optimistic estimate of $\sigma_F(E)$, i.e. the Becker et al value, with the assumption of 100% spin polarization. Thus, we assume $\sigma_{F0} = 1.8\times10^{-24}$, and we round off E_0 to ~600 keV. None of our conclusions are sensitive to the exact value of $\sigma_F(E)$ or E_0.
- $\nu_F = n_B v_0 \sigma_F(E_0)$ is the peak rate at which a proton undergoes fusion. Here, $v_0 = 1.1\times10^9$ cm/sec is the proton velocity at energy E_0. This rate also assumes fuel that is optimally spin-polarized for fusion.
- ν_{jk} is the effective momentum exchange collision frequency for species j, due to multiple Coulomb collisions with particles of species k.
- $\nu_{jk,E}$ is the rate of energy change for species j due to Coulomb-collisional energy exchange with species k.
- $\tau \equiv \nu^{-1}$ is the characteristic time for any process.

3. Collisional Processes

RBM have commented that a complete self-consistent analysis of the collisional dynamics of the CBFR would be a difficult computational task, given all of the collisional processes, driving forces, sources, sinks, and the complexities of the magnetic

confinement geometry. We agree with this assessment. However, it is relatively simple to perform a rigorous analysis of each of the collisional effects separately. We present just such an analysis in this section. Since RBM assume that the equilibrium can be maintained at least long enough to allow fusion of the protons resident in the system (in fact, their ultimate hope is to maintain a steady state),[19] we shall assume that in each scattering process, the *scatterers* present a steady state distribution. In particular, we shall begin with RBM's assumption[6] that the proton and ^{11}B constituents each form a cool beam, with relative energy ~600 keV between the two beams. We shall conclude that an equilibrium of the type proposed by RBM cannot be sustained for a time long enough to allow fusion gain, because any one of several collisional processes dissipate the assumed CBFR equilibrium on much shorter time scales.

A. Time Scales for Fusion and Collisional Processes

Table 1. Proton collision rates

Symbol	Description	Rate (sec^{-1})
ν_F	Fusion rate at resonance (polarized fuel)	$2.0\times10^{-15}n_B$
ν_{pB}	p-B momentum exchange collisions	$7.4\times10^{-14}n_B$
$\nu_{pB,E}$	p-B energy exchange collisions	$1.4\times10^{-14}n_B$
ν_{pp}	p-p collisions (momentum or energy exchange)	$7.2\times10^{-13}n_p\ (10\ \mathrm{keV}/T_p)^{3/2}$
ν_{pe}	Stopping of protons by p-e collisions	$2.6\times10^{-14}n_e\ (10\ \mathrm{keV}/T_e)^{3/2}$

Table 2. Boron collision rates

Symbol	Description	Rate (sec^{-1})
ν_{Bp}	Acceleration of boron by B-p collisions	$6.7\times10^{-15}n_p$
ν_{Be}	Acceleration of boron by B-e collisions	$5.9\times10^{-14}n_e(10\ \mathrm{keV}/T_e)^{3/2}$

The rates for protons to undergo various collisional processes[20] are shown in Table 1. Note that densities are in units of cm^{-3}. We notice immediately that the fusion reaction rate ν_F is much slower than the relevant rates for dissipative collisional processes. This is a red flag – especially for a fusion scheme that depends on colliding beams. For the p-^{11}B reaction, the peak fusion rate (per proton, using the most optimistic

estimate of the cross-section,[17] and assuming 100% polarized fuel at precisely the optimum collision energy $E_0 \cong 600$ keV for fusion) is $\nu_F = n_B\sigma_{F0}v_0 = 2.0\times10^{-15}n_B$ sec^{-1}, where σ_{F0} is the peak fusion cross-section and $v_0=1.1\times10^9$ cm/sec is the relative p-^{11}B velocity at energy E_0. However, the collision frequency for p-^{11}B momentum exchange scattering, i.e. for 90° scattering of the protons, is[20] $\nu_{pB} = 7.4\times10^{-14}n_B$ sec^{-1}, 37 times faster than the fusion rate ν_F. Momentum-exchange scattering has the primary effect of isotropizing the proton beam. This destroys its assumed beamlike velocity distribution, and leads to spatial spreading so that the protons are no longer confined to a tight annulus as assumed by RBM. Momentum-exchange p-^{11}B collisions also exert a drag on the boron beam, with the effective collision frequency $\nu_{Bp} = (n_pm_p/n_Bm_B)\nu_{pB}$ shown in Table 2. The consequent acceleration of the boron beam, which is over an order of magnitude (depending on the value of n_p/n_B) faster than fusion, detunes the assumed resonance at energy E_0. Proton energy loss due to p-^{11}B collisions proceeds at a rate $\nu_{pB,E} = 1.4\times10^{-14}n_B$ sec^{-1}, slower than ν_{pB} by about twice the mass ratio m_p/m_B, but still faster than ν_F by a factor ~ 7. This process slows down the protons, thereby additionally detuning the assumed resonance at energy E_0. The effects of p-^{11}B Coulomb scattering on the protons scale in the same way as the fusion rate (proportional to n_B), and thus these rates are faster than the fusion rate by the same large factor, irrespective of the choice of the density ratio n_p/n_B. In addition to the p-^{11}B scattering processes, there are various dissipative processes involving collisions between other species. The proton-proton Coulomb scattering frequency is[20] $\nu_{pp} = 7.2\times10^{-13}n_p\,(10\text{ keV}/T_p)^{3/2}$. For all scenarios considered by RBM for CBFR, $\nu_{pp} >> \nu_F$. Proton-proton collisions thermalize the protons in their own frame of reference, i.e. these collisions convert a spread in proton momenta (which results from p-^{11}B momentum scattering) into a proton energy spread, further degrading the fusion resonance. Proton-electron and boron-electron collisions are also faster than the fusion rate, for typical CBFR scenarios. These collisions indirectly couple the protons to the boron, leading to a friction that can stop the relative streaming of the ion beams, heat the electrons, and also influence the ion beam temperatures.

We shall now discuss these collisional processes in more detail. We also discuss the possibility of introducing driving forces, sources and sinks that can overcome the dissipation and sustain the equilibrium.

B. Proton-^{11}B Coulomb Scattering

The velocity distribution of p and ^{11}B is illustrated in Fig. 3a, for the typical "colliding-beam" equilibrium envisioned for the CBFR.[6] In Fig. 3, the horizontal axis

schematically represents the azimuthal velocity v_φ, which is the direction of beam flow. The vertical axis in this 2-D picture represents the two dimensions normal to the flow, v_r and v_z. The protons and the ^{11}B are each assumed to constitute a cold beam, i.e. the spread in velocities of each component is small compared to the relative azimuthal velocity between the two beams.

In this subsection, we consider the effects of p-^{11}B scattering on the proton distribution, and to do this we shall construct our picture in the frame of reference where the boron component is stationary, i.e. we shall use the variable $\mathbf{u} \equiv \mathbf{v} - \mathbf{V}_B$ to specify a proton's velocity. To simplify the discussion, let us assume that the boron beam actually has zero temperature. This is the most favorable situation for the CBFR, and finite boron temperature only makes matters worse. The nature of the scattering process is easy to understand intuitively. Because the mass ratio $m_B/m_p=11$ is large, the primary effect on the protons of multiple Coulomb collisions with the ^{11}B is to scatter the protons through a substantial angle. Energy transfer from the protons to the ^{11}B is slower by a factor $2m_p/m_B = 0.18$. Thus, to lowest order in the mass ratio, the Fokker-Planck equation reduces to simply diffusion of the proton velocity on a constant-u spherical shell in velocity space, as shown in Fig. 3b. To the next order in $2m_p/m_B$, the energy transfer between species results in a friction that draws the velocity of each proton toward the mean boron velocity $\mathbf{V}_B$.

We prove in the Appendix that this picture is rigorously correct: for scattering of protons off *any* cold species k, the Fokker-Planck equation reduces exactly to diffusion on a constant-energy sphere, plus friction; there is no diffusion in the proton energy $\frac{1}{2}m_p u^2$. According to Eqs. (A10,14), the Fokker-Planck equation for p-k scattering can be written in spherical velocity coordinates (u,θ,ψ) as

$$\frac{\partial f_p}{\partial t} = -\frac{1}{u^2}\frac{\partial}{\partial u}u^2 \tilde{F} f_p + \frac{D_\perp}{2u^2}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\psi^2}\right)f_p, \tag{3}$$

where the dynamical friction in spherical coordinates is

$$\tilde{F} = -\frac{4\pi n_k e^4 Z_k^{\ 2}}{m_p^{\ 2} u^2}\frac{m_p}{m_k}\lambda \equiv -\nu_{pk}\frac{m_p}{m_p + m_k}u, \tag{4}$$

the transverse diffusion coefficient is

$$D_\perp = \frac{4\pi n_k e^4 Z_k^{\ 2}}{m_p^{\ 2} u}\lambda \equiv \nu_{pk}\frac{m_k}{m_p + m_k}u^2, \tag{5}$$

and λ is the Coulomb logarithm. Note that these velocity coordinates are chosen so that the origin $u = 0$ is at the scatterer (i.e. boron) velocity, and the polar axis is along the difference in the beam fluid velocities, $\mathbf{V}_p - \mathbf{V}_k$.

The relative magnitude of the friction and transverse diffusion coefficients depends on the mass ratio m_p/m_k. For scattering of a light species (e.g. protons) off a heavy species (e.g. ^{11}B), Eqs. (4,5) show that transverse diffusion dominates, as stated above. If (for the moment) we neglect the friction term, the solution to Eq. (3) may be written in the form

$$f(\mathbf{v},t) = \frac{\delta(u-u_0)}{4\pi v^2} g(\theta), \tag{6}$$

where the cold beam initial condition is $g(\theta,0) = 2\delta(\theta^2)$ and $g(\theta,t)$ is the solution to

$$\frac{\partial g}{\partial t} = \frac{D_\perp}{2u^2} \frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \sin\theta \frac{\partial}{\partial\theta} g. \tag{7}$$

Thus the proton distribution diffuses on the surface of the constant-u sphere, over a time scale $\tau_{pB} \equiv \nu_{pB}^{-1}$, as shown in Fig. 3b. Note that this diffusion process steadily increases the proton temperature T_p, as defined in Eq. (2), but nonetheless all protons have the same energy $\frac{1}{2} m_p u^2$, as seen in the boron frame of reference. In an anisotropic, non-Maxwellian plasma, characterizing a species by a single "temperature" does not necessarily convey the same meaning as is usual in thermodynamics.

Thus, if p-^{11}B collisions were the only consideration, the constant-u spherical shell would fill in completely, as shown in Fig. 3c, at a time of order $2\tau_{pB}$ which is less than 3% of the time $\tau_F \equiv 1/\nu_F$ required for fusion. The resulting isotropic distribution function would no longer be a "beam" distribution, and it is incompatible with a thin-annulus proton equilibrium, or indeed with any rapidly rotating rigid-rotor-type distribution. Thus, the p-^{11}B scattering process is lethal to an equilibrium of the type assumed by RBM. However, the situation is made even worse by a peculiarity of the CBFR field-reversed configuration. In this magnetic confinement geometry, protons are confined only if they rotate azimuthally in the "diamagnetic direction." The coordinates may be defined so that this is the direction with $v_\varphi > 0$. Protons rotating in the opposite direction ($v_\varphi < 0$) promptly escape from the ends of the device.[21] As the protons diffuse to fill the energy shell, any protons scattered into the left hemisphere in Figs. 3b,c (i.e negative v_φ or $\theta > \pi$) are lost. Thus the diffusion process does not actually proceed to the point of uniformly filling the energy shell. Instead, about 97% of the protons are lost by

scattering into the negative-v_φ hemisphere before fusion takes place. Since the energy invested in proton acceleration (taking into account the imperfect efficiency of proton acceleration) amounts to at least 10% of the fusion output energy *if every proton fuses*, this in itself makes it clear that the CBFR cannot provide a net fusion gain.

C. Effect of ^{11}B-p Collisions on the Boron Beam

We have previously offered the argument of Sec. 3B in a comment on Ref. 1 which was sent to *Science*.[13] RBM responded as follows:[22]

> "These conclusions are inconsistent with conservation of momentum and energy that must be satisfied. Due to collisions, protons slow down and boron speeds up. From the conservation of momentum
>
> $$n_p m_p V_p + n_B m_B V_B = (n_p m_p + n_B m_B) V_c,$$
>
> with
>
> $$V_c = \frac{n_p m_p}{n_p m_p + n_B m_B} V_p + \frac{n_B m_B}{n_p m_p + n_B m_B} V_B \cong V_p$$
>
> because $n_B m_B / n_p m_p << 1$ and $V_B << V_p$. The final velocity of protons is only a little less than the initial value. . .The protons 'isotropize' as noted by Lampe and Manheimer, but in a frame of reference moving with the large azimuthal velocity V_c so that the number of protons lost because they rotate in the wrong direction is negligible. The energy loss of protons is much less than 600 keV."

In essence, RBM object to our treatment of the boron as a steady-state scatterer species with a constant fluid velocity V_B. To clarify matters, we restate the thinking underlying our calculations. The calculations of RBM all assume – indeed the CBFR concept only makes sense for p-^{11}B if one assumes – that there is an equilibrium in which the p and the ^{11}B each form a cold beam, with relative energy ~ 600 keV between the beams. Our objective is to test whether this assumed steady state can persist for a fusion time. Since it is not easy to do a complete self-consistent calculation of the evolution of all of the species, including all of the scattering processes, we begin by accepting the steady state assumption, as regards the scatterers in each scattering process, and address the question: If the scatterers constitute the required cold beam, is it possible for the scattered species to maintain its cold-beam distribution? In general, the answer is that it

is not. The required steady state is destroyed by whichever process turns out to be the fastest.

With this preamble, let us consider the effect of ^{11}B-p collisions on the boron. Our Eqs. (A10) show that the effect of p-^{11}B collisions on the boron is primarily a frictional drag; diffusion (i.e. heating) of the ^{11}B is slower by the mass ratio m_p/m_B. The time scale for friction to accelerate the boron to the proton beam velocity is (by conservation of momentum) $\tau_{Bp} = \tau_{pB}\, \rho_B/\rho_p$, where $\rho_B \equiv n_B m_B$ is the boron mass density and $\rho_p \equiv n_p m_p$ is the proton mass density. In much of the work of RBM, the parameters are chosen so that $\rho_B > \rho_p$, in which case proton scattering is faster than boron acceleration. But for the parameters cited cited in Ref. 1, n_p=400n_B and therefore $\rho_p >> \rho_B$. In this situation, acceleration of the boron beam is the faster process, as stated by RBM. The center-of-mass velocity V_c is indeed close to the initial proton velocity V_p, and the system will evolve to a state where $V_B = V_p = V_c$ over the time scale $\tau_{Bp} = 1.5\times10^{14} n_p^{-1} = 3.7\times10^{11} n_B^{-1}$, which is *over three orders of magnitude* faster than the proton fusion time τ_F. Thus, for this particular choice of parameters, we agree with the conclusion stated by RBM that the protons will not end up with a large velocity spread. However, the consequences of boron acceleration are even more devastating for the CBFR concept: the protons and ^{11}B end up as co-rotating beams, rather than colliding beams. The temperature of each of the beams is small compared to the 600 keV required for efficient p-^{11}B fusion (see Fig. 1), and virtually all encounters between protons and ^{11}B nuclei occur with relative energy << 600 keV. Consequently, *there will be virtually no fusion reactions at all.* For the CBFR to make any sense at all[6] for p-^{11}B, there must be some mechanism (as yet unspecified) that maintains the protons and boron as separate colliding beams with relative energy close to 600 keV. If the boron is somehow maintained at a constant rotation velocity, then our previous argument holds, and proton diffusion leads to isotropization about the boron velocity.

D. Energy Loss Effects Due to p-^{11}B Scattering

In the previous section, we have seen that the frictional acceleration of the boron beam, due to p-^{11}B collisions, can be important when $\rho_p > \rho_B$. Up to this point, we have neglected the energy loss of the protons due to p-^{11}B scattering, as it is less rapid than the pitch-angle scattering. However, if it is assumed that V_B is maintained at a constant value, the exact solution of the Fokker-Planck equation (3) for the proton velocity distribution, including both the friction and diffusion terms due to p-^{11}B scattering, is of the form

$$f_j(u,\theta,\varphi,t)=\frac{1}{4\pi v^2}\delta\big(u-u_0(t)\big)g(\theta,t), \tag{8}$$

where $u_0(t)$ is given by

$$\frac{du_0}{dt}=\tilde{F}, \tag{9}$$

$\tilde{F}$ is the friction coefficient of Eq. (4), and $g(\theta,t)$ is the solution to Eq. (A18) for diffusion on the collapsing spherical shell. Because $\tilde{F}$ is small, g is nearly identical to the diffusion Green's function defined in Eqs. (6,7). Thus, the proton distribution spreads out diffusively on the spherical shell, while the proton energy slowly decreases due to friction, i.e. the shell as a whole collapses down self-similarly toward V_B because of the friction. At any given time all protons have the same energy, i.e. the shell does not thicken, as there is no diffusion in proton energy. (Recall that this is a consequence of the assumption that the boron is cold). The time scale for the loss of all of the proton energy is $\nu_{pB,E}^{-1} \approx (m_B/2m_p)\nu_{pB}^{-1}$, which is still about seven times faster than the fusion timescale. Since the energy loss rate is rapid compared to the fusion rate, the relative energy between the protons and boron cannot be maintained at the resonant value 600 keV which is necessary for efficient p-^{11}B fusion. Indeed, the fusion rate falls off substantially when the protons lose only a quarter of their energy.

Once the angular direction of the proton velocities has been spread out on the energy shell, it is difficult to imagine any mechanism that could reaccelerate the protons to maintain the resonant energy. Any force that acts equally on all protons (e.g. an electric field), will merely displace the spherical shell as a whole, rather than expanding the shell as would be necessary to restore the resonance.

E. Maintenance of the Velocity Separation Between p and ^{11}B Beams

Irrespective of whether ρ_B/ρ_p is large or small, p-^{11}B friction brings $\mathbf{V}_p$ and $\mathbf{V}_B$ together in a time small compared to τ_F. If there are to be "colliding beams," there must be other forces that maintain the average velocities of each of the two beams. It is difficult to see how this could be done, but let us assume there is some such force active on the protons. The situation is then analogous to ordinary Ohmic heating, in which an electric field acts on electrons to maintain a constant electric current, despite friction due to electron-proton pitch-angle scattering. In the present case, the protons are the light species, and boron is the heavy species. As in ordinary Ohmic heating, the force must

supply energy to the protons at a rate $\frac{1}{2}\nu_{pB}n_p m_p(V_{p\varphi} - V_{B\varphi})^2$, which is about four times greater than the fusion power. This energy is dissipated as proton temperature, i.e. through the transverse velocity diffusion process discussed in the previous section.[23] So for this reason as well, the CBFR cannot possibly reach a steady state in which there is net fusion gain.

In several of their progress reports, and in response to comments by ourselves and by A. Carlson,[11] RBM argue that the separation between the azimuthal rotation velocities of the p and ^{11}B beams can be maintained by magnetic forces, arising from the radial cross-field drift of the protons. This argument is not viable. If the proton azimuthal velocity is assumed to be at steady state, with the proton-boron friction balanced by the $\mathbf{v}_r \times \mathbf{B}_z$ magnetic force, then it is necessary that

$$\frac{eV_rB}{c} = \nu_{pB}m_pV_\varphi\,, \tag{10}$$

i.e.

$$V_r = \frac{\nu_{pB}}{\Omega_p}V_\varphi = 8.5\times10^{-9}\,\frac{n_B\,(\mathrm{cm}^{-3})}{B\,(\mathrm{gauss})}\ \mathrm{cm/sec}\,, \tag{11}$$

where B is the magnetic induction and Ω_p is the proton cyclotron frequency. Taking B = 10^5 G, a characteristic value for the RBM scenarios, we find that the required radial velocity is $V_r \approx 8.5\times10^{-14} n_B$ cm/sec. Over a fusion time $\tau_F = 5\times10^{14} n_B^{-1}$ sec, a proton radial velocity V_r of this magnitude would lead to radial proton motion 43 cm. Since the proton ring is assumed by RBM to be only a few cm wide in all models of the CBFR, this would constitute radial loss of protons at a rate considerably exceeding the fusion rate. This loss mechanism was also mentioned by Goldston[8] in his letter to *Science*.

F. Effect of Boron Temperature

Up to this point, we have assumed that the boron beam is cold. But collisional interactions will heat the boron beam as well. Scattering of protons off *warm* ^{11}B differs only in that a proton energy diffusion term is also present in the Fokker-Planck equation, which in effect increases the thickness of the spherical shell on which the proton velocity distribution lies. This is a mechanism of temperature equilibration between species, operating in addition to the proton heating effects previously discussed, which are due to the streaming energy between the beams. Thus effects due to finite boron temperature only add to the proton temperature, and further detune the resonance.

G. Proton-Proton Scattering

As we have seen, the effect of p-^{11}B scattering on the protons is primarily to isotropize the protons about the boron velocity $\mathbf{V}_B$, and somewhat more slowly to extract energy from all protons at the same rate. Thus on the rapid time scale τ_{pB}, p-^{11}B collisions spread the protons out on an energy shell with characteristic velocity spread $v_p \sim |\mathbf{V}_p-\mathbf{V}_B|$, but leave all of the protons with pretty much the same energy, with respect to the boron. However, we must also consider proton-proton scattering, which drives the proton distribution toward an isotropic Maxwellian, centered on the mean proton velocity $\mathbf{V}_p$, and with thermal velocity v_p. The proton-proton momentum scattering rate is $\nu_{pp}=7.2\times10^{-13}n_p(10\ \mathrm{keV}/T_p)^{3/2}$; it may be larger or smaller than the p-^{11}B collision rate depending on T_p and (n_p/n_B), but ν_{pp} is orders of magnitude larger than the fusion rate ν_F in all CBFR designs considered by RBM. If $\nu_{pp} < \nu_{pB}$, proton-proton scattering modifies the spherical-shell proton velocity distribution, by filling in and spreading out the constant-energy shell, and over a longer time scale ν_{pp}^{-1} driving it toward Maxwellian. If $\nu_{pp} >> \nu_{pB}$, which is the more usual case for CBFR, proton-proton collisions keep the proton distribution close to a drifted Maxwellian at all times, with proton-boron collisions steadily increasing T_p. Since both $\nu_{pp} >> \nu_F$ and $\nu_{pB} >> \nu_F$, the proton streaming energy will in all cases thermalize long before the fusion time ν_F^{-1}, thereby totally obliterating the energy resonance.

H. Ion-Electron Collisions

In the CBFR scenario, the proton kinetic energy in the electron rest frame, $E_{pe}\equiv (m_p/2)(\mathbf{V}_p-\mathbf{V}_e)^2$, is much larger than the electron temperature T_e, but the electron thermal velocity v_e is much larger than the proton-electron relative streaming velocity $|\mathbf{V}_p-\mathbf{V}_e|$. In this regime, the effect of ion-electron collisions on the ions is very simple: the Fokker-Planck equation for the protons reduces to simply a friction

$$\mathbf{F}=\nu_{pe}(\mathbf{v}-\mathbf{V}_e) \qquad (12)$$

on the protons. The diffusion terms in the Fokker-Planck equation are smaller by a factor T_e/E_{pe}, and can be neglected. The collision frequency for this process is $\nu_{pe}=2.6\times10^{-14}(10\ \mathrm{keV}/T_e)^{3/2}n_e$, independent of the velocity of a particular ion. These well-known facts are proven explicitly in the Appendix. Thus the effect of electron scattering on the protons is to draw each proton's velocity toward the mean electron velocity $\mathbf{V}_e$. Similarly, the effect

of electron scattering on the boron is to draw each ^{11}B nucleus's velocity toward the mean electron velocity $\mathbf{V}_e$, in this case at a rate ν_{Be} which is larger than ν_{pe} by a factor $(Z_B^2/A_B)=25/11$. These rates are faster than the fusion rate ν_F for most CBFR scenarios, and for those cases where T_e is not very large, or where $n_e >> n_B$, they can be orders of magnitude larger.

Let us consider the effects of these collisions. The electron inertia is always small compared to the ion inertia, and thus if there were only one ion beam (say the proton beam), the electrons would very quickly be speeded up to the proton velocity $\mathbf{V}_p$, with very little influence on the protons. However the electrons are also subject to collisions with the ^{11}B and thus they settle down to a fluid velocity $\mathbf{V}_e$ intermediate between $\mathbf{V}_p$ and $\mathbf{V}_B$. Let us assume that $\mathbf{V}_e$ is in a steady state. The effect of p-e collisions on the protons is thus to draw each proton toward velocity $\mathbf{V}_e$. Similarly, ^{11}B-e collisions draw each ^{11}B nucleus toward $\mathbf{V}_e$. Thus the electrons act as an intermediary that couples the protons and boron through a friction characterized by the collision frequencies ν_{pe} and ν_{Be}. This friction is in addition to the friction due to direct p-^{11}B collisions. In most of the CBFR scenarios that have been discussed by RBM, the electron-mediated friction is fast compared to the fusion rate, and thus effectively prevents maintenance of the required beam equilibrium. In the scenario presented in the *Science* article[1], where $n_p = 400n_B$ and thus $n_e = 405n_B$, and $T_e = 20$ keV, we find the rates to be $\nu_{pe} = 9\times10^{-15}n_e = 1.8\times10^3\nu_F$ and $\nu_{Be} = 2.1\times10^{-14}n_e = 4.2\times10^3\nu_F$; the beams are virtually stopped in their tracks by electron friction.

RBM have argued[1] that proton-electron collisions lead to significant reduction of the proton beam temperature. It is true that electron collisions can have a cooling effect on the protons, but this is merely an incidental effect of the friction that the electrons exert on the protons: every proton is drawn toward $\mathbf{V}_e$, and thus according to Eq. (12) the spread in the proton velocity distribution (as seen from the electron mean velocity $\mathbf{V}_e$) contracts by the same factor as the reduction in $|V_p - V_e|$. If the CBFR is to succeed, parameters must be chosen so that $\nu_{pe} < \nu_F$; otherwise, proton-electron friction will stop the proton beam. However, heating of protons due to p-^{11}B pitch-angle scattering proceeds at the frequency ν_{pB}, which we have seen to be much faster than ν_F. Thus, electron cooling cannot prevail. Additionally, it should be noted that for very non-thermal velocity distributions, "cooling" in the sense of reducing the spread in $\mathbf{v}-\mathbf{V}_e$ is not necessarily equivalent to reduction of the spread in the relative proton-boron velocity $\mathbf{v}$-$\mathbf{V}_B$, nor to reduction of the temperature T_p defined in Eq. (2). Consider, for example, a proton distribution spread on a constant-$|\mathbf{v}-\mathbf{V}_B|$ shell, such as would be produced by p-^{11}B

collisions alone, as seen in Fig. 3b. Since $\mathbf{V}_e \neq \mathbf{V}_B$, the electron friction draws each point on the proton shell toward a different velocity than the boron friction. As a result, the shell is distorted so that it is no longer a spherical shell centered on $\mathbf{V}_B$. This effect *increases* the spread in proton energies as seen by the boron beam, i.e. in $E_{pB} \equiv \frac{1}{2} m_p \left(\mathbf{V}_p - \mathbf{V}_B\right)^2$.

As a final comment on the subject, we note that in most cases it is necessary that the electrons be somehow maintained at a large temperature T_e, in order to reduce the friction rate ν_{pe}. For example, in their most recent progress report,[14] RBM propose a scenario in which T_e = 165 keV. When protons scatter off *warm* electrons, a proton energy diffusion term is present in the Fokker-Planck equation in addition to the friction term, which increases T_p in addition to the effects previously mentioned.

Collisional Effects: Summary Statement

In Sections 3A–H we have listed at least eight separate dissipative effects, due to ordinary Coulomb collisions, each of which detunes the resonance and/or destroys the "colliding beam" equilibrium, on a time scale fast compared to the fusion time scale. These processes are:

1. Isotropization of the proton beam due to p-^{11}B scattering.
2. Loss of protons scattered into $v_\varphi < 0$.
3. "Ohmic" heating of the protons, a consequence of any force that maintains the required p-^{11}B streaming velocity in the presence of p-^{11}B scattering.
4. Radial escape of protons, if the radial drift velocity is assumed large enough to maintain the required p-^{11}B streaming velocity.
5. Detuning of the resonance due to boron acceleration, resulting from ^{11}B-p friction.
6. Detuning of the resonance due to proton energy loss, resulting from p-^{11}B friction.
7. Stopping of the relative streaming of the p and ^{11}B beams, due to electron friction.
8. Thermalization of the proton momentum spread, due to p-p collisions.

Each of these processes increases the entropy. For successful operation of the p-^{11}B CBFR, additional collisional or other effects would have to be found which cancel, to within better than 90% (and in some cases 99%), all of these effects of the momentum and energy exchange collisions. Eight mutually consistent miracles is a lot to ask for, even in the fusion business!

In their letters to *Science*, Nevins,[10] Carlson,[11] and Rider [12] all point out that the protons and ^{11}B cannot be maintained as cold colliding beams, because of these various ion-ion scattering processes. In their response to Rider,[24] Rostoker and Binderbauer appear to concede this point. They state:

> "The ion-ion collision term is certainly a problem, particularly if the investigation is not device specific, but is evaluated with generic formulae. There are compensating terms for the CBFR.
>
> The problem of ion-ion collisions can be avoided with a different mode of operation where both beams have the same average velocity and temperature. Then only ion-electron collisions need to be considered."

Obviously, this makes no sense for a p-^{11}B reactor, where the fusion reaction rate is strongly peaked at ~600 keV, the energy invested in accelerating the beams exceeds 10% of the fusion output even if every proton fuses, and the maximum possible thermal fusion gain is only a little larger than unity. For p-^{11}B, RBM themselves state[6] that, "it is necessary to maintain the proton and boron beams at an average energy difference of 580 keV. In addition, the temperatures of the beams must be substantially less than 140 keV."

4. Heating of Protons via Proton-Boron Collisions

In their most recent ONR progress report,[25] and more forcefully in a widely circulated letter responding to our criticism,[26] RBM concede that the heating or cooling of the beams is a crucial issue, and then argue that collisional scattering of the protons by the streaming boron beam *cools* the protons, i.e. reduces the temperature T_p defined by Eq. (2). In their response[27] to Nevins,[10] RBM claim that collisional scattering of the electrons by the streaming boron beam *cools* the electrons. These are amazing claims. It should be very clear that scattering a cold beam of light particles (protons or electrons) off a cold beam of heavy particles (^{11}B) can only increase the temperature of the light beam. The analogy that comes to mind is scattering a well-aligned stream of tennis balls off a distribution of bowling balls. Obviously, in the frame of reference in which the bowling balls are stationary, the well-aligned kinetic energy of the tennis balls is quickly transformed into random kinetic energy, i.e. temperature.

In this section we hope to lay this question to rest. We derive an explicit equation for the time evolution of the proton temperature, based on the Vlasov / Fokker-Planck

equation. We include proton-boron collisions, proton-proton collisions, a driving electric field, and a source and sink of protons. The calculation, which is done essentially exactly, clearly shows that the effect of p-^{11}B collisions is to heat the protons. We then retrace the derivation offered by RBM, to elucidate the errors that led them to conclude that the protons are cooled by p-^{11}B collisions.

This section includes more mathematical detail than most of the paper. Since proton heating has been a point of controversy between us and RBM, we have tried to present the derivations in enough detail to make them accessible to the general reader, and hopefully to resolve any disagreements.

Our treatment begins with the kinetic equation controlling the normalized proton velocity distribution $f_p(\mathbf{v},t)$,

$$\frac{\partial}{\partial t}\left(n_p f_p\right)+\frac{n_p e\mathbf{E}}{m_p}\cdot\frac{\partial f_p}{\partial \mathbf{v}} = -n_p\frac{\partial}{\partial \mathbf{v}}\cdot\mathbf{F}f_p+\tfrac{1}{2}n_p\frac{\partial^2}{\partial \mathbf{v}_\perp \partial \mathbf{v}_\perp}:\mathbf{D}_\perp f_p + n_p\left.\frac{\partial f_p}{\partial t}\right|_{pp} \qquad (13)$$

$$+\dot{n}_1\delta(\mathbf{v}-\mathbf{v}_1)-\overline{\nu}_F n_p\delta(\mathbf{v}-\mathbf{v}_2)\,.$$

The second term on the LHS is the standard Vlasov term representing the effect of an electric field. We have in mind that $\mathbf{E}$ is primarily an azimuthal electric field E_φ which may be present to drive the proton streaming. The first two terms on the RHS are the Fokker-Planck friction and diffusion terms representing p-^{11}B scattering, as discussed in the Appendix. These terms are the focus of our interest here. We note that the diffusion tensor $\mathbf{D}$ is purely transverse for scattering off a cold boron beam, as proven in the Appendix. We have also included, for completeness, a collision integral representing p-p collisions, although this term contributes nothing to the fluid equations. The last two terms represent the source of new protons (the injected proton beam), and the sink of protons due to fusion events. These terms are also included for completeness, although they make only a small contribution to the proton temperature equation. The quantity $\overline{\nu}_F$ is an average fusion rate, which will necessarily be somewhat smaller than the peak fusion rate ν_F (at energy E_0) defined previously. We assume that the protons are injected at a well-defined azimuthal velocity $\mathbf{v}_1$ (a very realistic assumption), and that fusion reactions occur at a well-defined velocity $\mathbf{v}_2$ (an unrealistic assumption that is made only to simplify the presentation). Our treatment assumes that both beams are uniform in the azimuthal direction φ (the streaming direction). We also neglect inhomogeneities in the radial and axial directions, as they are not relevant to the present discussion. A more rigorous formulation would express the results as a radial average, as is done by RBM in their analysis. This complicates the notation, but does not change the results.

We proceed to derive fluid equations by taking velocity moments of Eq. (13). This procedure can be carried out rigorously, because the beams have been taken to be homogeneous. We first take the velocity integral of Eq. (13), to arrive at a density continuity equation. This equation is trivial:

$$\frac{dn_p}{dt} = \dot{n}_1 - \overline{\nu}_F n_p . \tag{14}$$

In steady state, the beam injection rate must equal the fusion loss rate, so that (14) reduces simply to

$$\frac{dn_p}{dt} = 0, \qquad \dot{n}_1 = \overline{\nu}_F n_p . \tag{15}$$

We shall assume Eqs. (15) are valid. We next take the moment $\int d^3\mathbf{v}\; m_p\mathbf{v}$, in order to derive the momentum equation:

$$m_p \frac{\partial}{\partial t}\int d^3\mathbf{v}\mathbf{v}f_p + e\mathbf{E}\cdot\int d^3\mathbf{v}\frac{\partial f_p}{\partial \mathbf{v}}\mathbf{v} = -m_p\int d^3\mathbf{v}\mathbf{v}\frac{\partial}{\partial \mathbf{v}}\cdot\mathbf{F}f_p + \overline{\nu}_F m_p(\mathbf{v}_1 - \mathbf{v}_2). \tag{16}$$

The Fokker-Planck diffusion term and the p-p collision term make no contribution, as these terms conserve total proton momentum. Using integration by parts, Eq. (16) reduces exactly to

$$m_p\frac{d\mathbf{V}_p}{dt} = e\mathbf{E} + m_p\int d^3\mathbf{v}\,\mathbf{F}f_p + \overline{\nu}_F m_p(\mathbf{v}_1 - \mathbf{v}_2) . \tag{17}$$

Finally, we take the moment $\int d^3\mathbf{v}\frac{1}{2}m_p\left(\mathbf{v}-\mathbf{V}_p\right)^2$ to derive a temperature equation. The p-p collision term makes no contribution, since this term conserves total proton energy. Using the definition (2) of T_p, and performing integration by parts for the Fokker-Planck terms (but no approximations), the remaining terms can be put in the form

$$\begin{aligned}\frac{3}{2}\frac{dT_p}{dt} = e\mathbf{E}\cdot\int d^3\mathbf{v}\left(\mathbf{v}-\mathbf{V}_p\right)f_p + m_p\int d^3\mathbf{v}\left(\mathbf{v}-\mathbf{V}_p\right)\cdot\mathbf{F}f_p + \tfrac{1}{2}m_p\int d^3\mathbf{v}Df_p \\ + \overline{\nu}_F m_p\left(v_1 - v_2\right)\left(\frac{v_1+v_2}{2} - V_p\right),\end{aligned} \tag{18}$$

where D is the trace of the Fokker-Planck diffusion tensor. We note that the first term on the RHS of (18) integrates to zero, i.e. the electric field makes no direct contribution to dT_p / dt. We now use the explicit forms for $\mathbf{F}$ and $\mathbf{D}_\perp$ obtained in Eqs. (A10) of the Appendix:

$$\mathbf{F} = -\nu_{pB}(\mathbf{v}-\mathbf{V}_B), \tag{19a}$$

$$D = 2\nu_{pB}\frac{m_B}{m_B+m_p}\left(\mathbf{v}-\mathbf{V}_B\right)^2, \tag{19b}$$

where ν_{pB} is the proton-boron collision frequency,

$$\nu_{pB} = \frac{4\pi n_B e^4 Z_B{}^2}{m_p\left(\mathbf{v}-\mathbf{V}_B\right)^3}\,\frac{m_B+m_p}{m_B}\lambda, \tag{20}$$

and λ is the Coulomb logarithm. The collision frequency ν_{pB} is a slowly-varying function of $|\mathbf{v}-\mathbf{V}_B|$, but $|\mathbf{v}-\mathbf{V}_B|$ is nearly the same for all protons in the cold beam. Thus we shall treat ν_{pB} as a constant. (This very reasonable approximation is also used by RBM in all of their calculations.) The momentum equation (17) then reduces exactly to

$$m_p\frac{d\mathbf{V}_p}{dt} = e\mathbf{E} - m_p\nu_{pB}\left(\mathbf{V}_p-\mathbf{V}_B\right) + \nu_F m_p(\mathbf{v}_1-\mathbf{v}_2), \tag{21}$$

while the temperature equation (18) becomes

$$\begin{aligned}\tfrac{3}{2}\frac{dT_p}{dt} = &-m_p\nu_{pB}\int d^3v\left(\mathbf{v}-\mathbf{V}_p\right)\cdot\left(\mathbf{v}-\mathbf{V}_B\right)f_p + \frac{m_p m_B}{m_B+m_p}\nu_{pB}\int d^3v\left(\mathbf{v}-\mathbf{V}_B\right)^2 f_p \\ &+\bar{\nu}_F m_p\left(v_1-v_2\right)\left(\frac{v_1+v_2}{2}-V_p\right),\end{aligned} \tag{22}$$

Collecting terms, Eq. (22) reduces to

$$\tfrac{3}{2}\frac{dT_p}{dt} = \nu_{pB}\frac{m_B m_p}{m_B+m_p}\left(V_p-V_B\right)^2 - 3\nu_{pB}\frac{m_p}{m_B+m_p}T_p + \bar{\nu}_F m_p\left(v_1-v_2\right)\left(\frac{v_1+v_2}{2}-V_p\right). \tag{23}$$

The first term of Eq. (23) represents heating of the protons due to p-^{11}B scattering. The CBFR requires $m_p(V_p-V_B)^2 \sim 1.2$ MeV $\gg T_p$. Thus the second term on the RHS is a small correction, and it is clear that the scattering process results in proton heating, not cooling. The last term is also a small correction, since the fusion rate $\bar{\nu}_F$ is much smaller than ν_{pB} and v_1-v_2 may be expected to be less than $|V_p-V_B|$.

We now retrace through Ref. 14 to try to understand how RBM arrive at the contrary conclusion that proton-boron scattering can cool the protons. The derivation begins in Appendix A of Ref. 14, on p. 9, and then continues in the main body of the paper, on p. 4. In Appendix A, the discussion is in terms of the effects of scattering on electron temperature, but on p. 4 the previous equations are applied to proton temperature. The justification for some of the steps in the derivation is not evident.

The RBM derivation begins with Eq. (4), p. 9, in Appendix A of Ref. 14, which we restate in our notation as

$$\frac{3}{2}\frac{dT_p}{dt} = m_p \int d^3\mathbf{v}\left[(\mathbf{v}-\mathbf{V}_p)\cdot\mathbf{F}+\tfrac{1}{2}D\right]f_p(\mathbf{v}), \tag{24}$$

[RBM use the notation $\langle\Delta\mathbf{v}\rangle$ for our **F**, and $\langle\Delta\mathbf{v}\Delta\mathbf{v}\rangle$ for our **D**. We have rewritten the equation for protons instead of electrons, and we have suppressed the radial averaging that is applied to every term and makes the notation look much more complicated.] Equation (24) is similar to our Eq. (18). The source and sink terms are not included, but they are not relevant to the present discussion. However, RBM then continue with:

"We note that if n_p and V_p are constant,

$$\frac{3}{2}\frac{dT_p}{dt} = e\mathbf{V}_p\cdot\mathbf{E}+m_p \int d^3\mathbf{v}\left[\mathbf{v}\cdot\mathbf{F}+\tfrac{1}{2}D\right]f_p(\mathbf{v})." \tag{25}$$

In going from Eq. (24) to Eq. (25), a term $-\mathbf{V}_p\cdot\mathbf{F}$ has been replaced by $e\mathbf{V}_p\cdot\mathbf{E}$. We can only speculate that the intention is to insert the effect of an electric field, and to reason that if a steady state exists the electric field must take the value $e\mathbf{E} = -\mathbf{F}$ [see Eq. (17)] so as to counterbalance the friction **F**. Since the proton flow is primarily in the azimuthal direction, **E** and **F** would also have to be vectors directed primarily in the azimuthal direction. If this substitution is made, the $e\mathbf{V}_p\cdot\mathbf{E}$ term must be regarded as an essential part of the effect of p-^{11}B scattering on dT_p/dt. However, this term is simply ignored throughout the remainder of RBM's Appendix A, up to the end of p. 10. The discussion is concerned entirely with the evaluation of the second term on the RHS of Eq. (25), which RBM seem to regard as a complete description of the p-^{11}B interaction. In this way, RBM finally arrive their Eq. (12), p. 10, which is (in our notation)

$$\frac{3}{2}\frac{\partial T_p}{\partial t} = -A\nu_{pB}m_p V_B\left(V_p - V_B\right) \tag{26}$$

where A is a coefficient that includes area-weighting. This is essentially the equation that is subsequently quoted in the main text of their paper [Eq. (9), p.4], and in subsequent letters, as indicating that the effect of p-^{11}B collisions is to cool the protons.

We do not understand the reasoning behind these steps, which do not constitute a correct treatment of the steady state nor of the electric field effect, and which are clearly

inconsistent with the derivation of Eqs. (18,22,23) directly from the Vlasov/Fokker-Planck equation (13). It should be clear just from its form that Eq. (26) cannot be correct: Temperature, as defined in Eq. (2) – the same definition used by RBM – is a quantity that is independent of the frame of reference of the calculation. Equation (26) does not have this property: on the RHS, the factor $(V_p - V_B)$ is frame-invariant, but the factor V_B is not. By merely doing the calculation in a different (rotating) frame of reference, the "cooling" effect can be converted to a heating effect, of arbitrary magnitude. If RBM had followed through with the calculation directly from Eq. (24) or (25), they would have arrived at an equation essentially equivalent to our Eq. (23). Indeed, they themselves, immediately after writing Eq. (26), comment (at the top of p. 11, Ref. 14) that

"If V_e was not assumed to be constant the result would be

$$\frac{\partial T_p}{\partial t} = -A\nu_{pB}m_p\left(V_p - V_B\right)^2 ." \tag{27}$$

Equation (27), which is equivalent to our Eq. (23), does have the correct property of frame-invariance, and is the correct result in steady state or in a transient situation.

Strangely enough, RBM do eventually return to the term $e\mathbf{V}_p \cdot \mathbf{E}$ in Eq. (25), but only to include the *radial* component $eV_{pr}E_r$, which is associated with the slow radial diffusive drift of the protons in a steady state burning plasma. This contribution is discussed on p. 11 of RBM's Appendix A, and also on pp. 4-5 of their main text. As noted by RBM, this radial term is small.

5. Other Issues Related to the p-^{11}B CBFR

In this section, we comment very briefly on some additional technological and scientific issues. Although these issues do not have the force of obvious show-stoppers, they do appear to raise serious concerns regarding the viability and utility of a CBFR reactor, particularly in regard to the naval shipboard power application, where compactness is of the essence.

A. Beam Technology and Size Constraints

The ONR reports[3] of RBM are largely concerned with the application of this fusion scheme as a power source for naval ships. For this application, minimization of

device size is crucial, and RBM devote considerable effort to this issue. In the *Science* article as well,[1] continued interest in the naval application is evident, and indeed Fig. 3 of Ref. 1 is intended to show the compact nature of a naval reactor. However, nowhere is there any discussion of the proton beam source, which is simply sketched in that figure as a box approximately $1 \times 2\,\mathrm{m} \times 2\,\mathrm{m}$ in size.

For purely steady state operation, neutral beam injection would be required. For the naval application, the RBM scheme requires 20 amps of neutral H at energy > 600 keV, i.e. an input beam power of > 12 MW, to produce a maximum fusion power of about 160 MW if every proton fuses. The preeminent existing program in neutral H beam development is being conducted in Japan, for the purpose of heating the JT 60-U Tokamak.[28] This facility currently delivers 2.5 MW of H atoms at 350 keV, and planned upgrades will bring it to 10 MW at 500 keV, comparable to the beam required for the CBFR. However, the volume required for neutral beam production and focusing is enormous, compared to the scale size of 2m sketched by RBM in Fig. 3 of Ref. 1. The sources are approximately 6m long, and because the beams are generated with very large diameter and must be focused down with good collinearity (5 mrad), the focusing length is approximately 18 m. There is no known neutral beam source technology which is close to the compactness needed for the naval application.

A more fundamental concern is that trapping of the neutral H beam in the CBFR plasma would appear to be very inefficient, because the proton ring envisioned by RBM presents a very small target. To be trapped in the plasma, a beam H atom must be ionized by interaction with the plasma. In the regime of interest, electron-impact is the predominant ionization mechanism, with an ionization cross-section[29] of about 10^{-18} cm^2 at $T_e = 20$ keV. (The charge-exchange cross-section is even smaller at proton energy 600 keV; extrapolation from Fig. 3.1 of Brown[30] indicates that it is about $5{\times}10^{-20}$ cm^2.) For electron density $n_e = 2{\times}10^{15}$ cm^{-3}, the mean free path for ionization of an H atom is thus ~500 cm. But for injection perpendicular to the magnetic field, the CBFR originally proposed with radius $r = 30$ cm and thickness $\Delta r = 4$ cm presents a target plasma at most 30 cm long along any chord, as shown in Fig. 4, and 15 cm on the average. Some of the more recent designs by RBM[3] envision a proton ring with radius up to 80 cm. Even if $\Delta r = 10$ cm, the average chord is only 40 cm, so that less than 10% of the beam will be trapped. Furthermore, the narrowest beam that can be produced by the JT 60-U facility (or any planned follow-on technology) is much wider than the maximum acceptance of the CBFR plasma. Thus neutral beam injection, using any reasonable extrapolation of today's technology, would not appear to be a viable driver for the p-^{11}B CBFR.

An alternative approach is repetitively pulsed injection of a neutralized ion beam (i.e. an ion beam accompanied by an equal and opposite charge density of electrons), with parameters chosen so that the ion plasma frequency exceeds the ion gyrofrequency. Then the neutralized ion beam can cross the vacuum magnetic field via the **E**×**B** drift, where **E** is the polarization field. The polarization field is shorted out in the dense plasma, and the beam is trapped.[31] The most powerful present-day repetitively pulsed ion beam sources, e.g. the RHEPP-II ion accelerator at Sandia,[32] produce a time-averaged current of only 0.1 amp, *two orders of magnitude below the CBFR requirement* for a 100 MW naval reactor, and a time-averaged power of 300 kW, a factor of 40 below the CBFR requirement. (However, the design voltage of 2.5 MV is three to four times greater than the requirement for CBFR.) This source occupies a volume of about 400 m^3, over two orders of magnitude larger than the ion accelerator sketched in Fig. 3 of Ref. 1. The most advanced proposed repetitively pulsed electron sources, e.g. the E-Scrub electron accelerator[33] proposed by SAIC (for which minimization of size was a prime objective), would occupy a volume of 500 m^3 for 12 MW of average power at voltage 800 kV. Electron sources are typically considerably more compact than ion sources. There is no known technology which can supply the rep-rated ion beams for the CBFR with the compactness envisioned for the naval application.

B. Experimental Evidence on Field-Reversed Ion Rings

There is a lengthy publication record on field-reversed configurations.[34,35] However, this body of work is concerned primarily with plasma-filled RFC's. The experimental evidence on RFC's powered by injected ion rings is scant, and it is not encouraging in regard to the prospects for maintaining a thin ring at the resonant energy necessary for the p-^{11}B reaction. Experiments were done in the 1980's at NRL[36] and Cornell[37]. In both cases, the ion ring spread out radially so that the thickness Δr became comparable to r. These experiments were not specifically designed to produce thin rings, but they do suggest that maintaining a thin ring may be very difficult.

C. Instabilities

As RBM noted in their response to Rider,[24] "The list of possible instabilities is endless." However, we shall simply comment here that, on general theoretical grounds one might expect that a thin field-reversed ion ring equilibrium, if it could somehow be produced, would be susceptible to major disruptions. After nearly 40 years of tokamak research, the problem of major disruptions still has not been solved. Tokamaks are low-beta devices, so it is only the poloidal field energy which is released in a disruption. In the CBFR, virtually all of the field energy could be released. For the 30 cm radius beam with a 40 cm inner wall, the magnetic free energy of the field reversed configuration (i.e., the difference between the magnetic energy in the properly configured CBFR, and the magnetic energy of a uniform solenoidal field with the same flux) is about 20 MJ (equivalent to about 5 pounds of TNT) per meter. This far exceeds the energy content (about 4 MJ per meter) of the thin ion ring which separates the reversed field regions. A single major disruption on this scale would most likely destroy the superconducting magnet.

6. Conclusions

The use of the p-^{11}B fusion reaction is alluring, because of the absence of neutrons and the possibility of direct conversion of fusion energy to electricity. These would be major advantages in a civilian power reactor, and they would be overwhelming advantages in a naval power source. Unfortunately, as has been noted by many authors,[5,8,9] the energy balance for p-^{11}B fusion is so marginal that virtually everything must work out perfectly to make net gain possible. In particular, to achieve substantial gain it would appear to be necessary to operate with colliding p and ^{11}B beams (as proposed by Rostoker, Binderbauer and Monkhorst) rather than with a thermal plasma, in order to exploit the resonance in the fusion cross-section at about 600 keV.

Unfortunately, for p-^{11}B there are many Coulomb collisional processes which are orders of magnitude faster than the fusion rate. We have pointed out eight processes, each of which has the effect of dissipating the cold-colliding-beam equilibrium by heating the beams and/or bringing their velocities together. There is no possibility of preserving the required equilibrium for long enough to allow a substantial fraction of the injected protons to undergo fusion, and therefore no possibility of achieving net gain from the CBFR. These arguments would appear to apply as well to any type of colliding-beam scheme for p-^{11}B fusion.

We have additionally pointed out that there is no reason to expect that the CBFR concept could be implemented in a compact form suitable for a naval power source, inasmuch as the technology for producing and injecting beams involves very large equipment. Some other difficulties, related to beam trapping efficiency and to plasma stability, have also been discussed in passing.

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Appendix: Analysis of the Fokker-Planck Equation

The Rosenbluth-MacDonald-Judd form of the Fokker-Planck equation,[38] which represents the effect of multiple Coulomb collisions in a plasma, is

$$\frac{\partial f_j}{\partial t} = -\frac{\partial}{\partial \mathbf{v}} \cdot \mathbf{F} f_j + \frac{1}{2} \frac{\partial^2}{\partial \mathbf{v} \partial \mathbf{v}} : \mathbf{D} f_j , \tag{A1}$$

where $\mathbf{F}(\mathbf{v})$ and $\mathbf{D}(\mathbf{v})$ are the dynamical friction and velocity diffusion tensor, given by

$$\mathbf{F} = \frac{4\pi n_k e^4 Z_k{}^2 Z_j{}^2}{m_j{}^2} \frac{m_j + m_k}{m_k} \lambda \frac{\partial}{\partial \mathbf{v}} \int d^3\tilde{\mathbf{v}} \frac{f_k(\tilde{\mathbf{v}})}{|\mathbf{v} - \tilde{\mathbf{v}}|}, \tag{A2a}$$

$$\mathbf{D} = \frac{4\pi n_k e^4 Z_k{}^2 Z_j{}^2}{m_j{}^2} \lambda \frac{\partial^2}{\partial \mathbf{v} \partial \mathbf{v}} \int d^3\tilde{\mathbf{v}} \, f_k(\tilde{\mathbf{v}}) |\mathbf{v} - \tilde{\mathbf{v}}| , \tag{A2b}$$

j is the species whose distribution function $f_j(\mathbf{v})$ is being followed, k is the scatterer species [Eqs. (A2) should be summed over various scatterer species], and λ is the Coulomb logarithm. If the scatterer distribution is isotropic about its own mean velocity $\mathbf{V}_k$, we have shown[39] that $\mathbf{F}$ and $\mathbf{D}$ can be reduced to the following forms:

$$\mathbf{F} = -\frac{16\pi^2 n_k e^4 Z_k{}^2 Z_j{}^2}{m_j{}^2 u^3} \frac{m_j + m_k}{m_k} \lambda \mathbf{u} \int_0^u d\tilde{v} \tilde{v}^2 f_k(\tilde{v}) , \tag{A3a}$$

$$D_{\parallel} = \frac{32\pi^2 n_k e^4 Z_k{}^2 Z_j{}^2}{3 m_j{}^2} \lambda \left[\frac{1}{u^3} \int_0^u d\tilde{v} \tilde{v}^4 f_k(\tilde{v}) + \int_u^\infty d\tilde{v} \tilde{v} f_k(\tilde{v}) \right], \tag{A3b}$$

$$D_{\perp} = \frac{16\pi^2 n_k e^4 Z_k{}^2 Z_j{}^2}{3 m_j{}^2} \lambda \left[\frac{1}{u^3} \int_0^u d\tilde{v} \tilde{v}^2 \left(3u^2 - \tilde{v}^2\right) f_k(\tilde{v}) + 2\int_u^\infty d\tilde{v} \tilde{v} f_k(\tilde{v}) \right], \tag{A3c}$$

where $\mathbf{u} \equiv \mathbf{v} - \mathbf{V}_k$, $D_{\parallel}$ is the parallel diffusion (the diagonal component of $\mathbf{D}$ along $\mathbf{u}$), and $D_{\perp}$ is the transverse diffusion (the diagonal component of $\mathbf{D}$ perpendicular to $\mathbf{u}$).

Scattering of Energetic Protons by Electrons

For proton scattering off electrons, the friction coefficient **F** is multiplied by a very large factor m_p/m_e, which is lacking in the diffusion coefficients. This leads one to expect friction to be dominant over diffusion. Looking at the situation more carefully, it becomes clear that friction dominates over diffusion if the proton kinetic energy is large compared to the electron temperature. The proof of this is as follows. With velocities referenced to the electron fluid velocity $\mathbf{V}_e$, the fractional change in proton velocity in a time interval Δt is

$$\frac{\Delta u}{u} = \frac{F\Delta t}{u}, \tag{A4a}$$

whereas the fractional spread in proton kinetic energy due to diffusion is

$$\frac{\Delta u^2}{u^2} = \frac{D\Delta t}{u^2}. \tag{A4b}$$

Evaluating Eqs. (A3), we find that in the limit of *very* high proton energy, where the proton velocity u exceeds the electron thermal velocity v_e,

$$\frac{\Delta u / u}{\Delta u^2 / u^2} = \frac{m_p}{2m_e} >> 1. \tag{A5}$$

and in the intermediate regime where

$$\tfrac{1}{2} m_p u^2 >> T_e, \quad \text{but } u < v_e, \tag{A6}$$

we find

$$\frac{\Delta u / u}{\Delta u^2 / u^2} = \frac{m_p u^2}{2T_e} >> 1. \tag{A7}$$

Thus, for scattering of energetic protons (i.e. protons with $mu^2 >> 2T_e$) by electrons, the diffusion terms in the Fokker-Planck equation are always small, and the Fokker-Planck equation reduces to simply a frictional drag on the protons. Furthermore, in the intermediate proton velocity regime where (A6) holds, (A3a) can integrated approximately, to give

$$\mathbf{F} = -\frac{4\sqrt{2\pi}\, n_e e^4}{3 m_p m_e} \left(\frac{m_e}{T_e} \right)^{3/2} \lambda \mathbf{u} \equiv \nu_{pe} \mathbf{u}. \tag{A8}$$

We see that in this regime the friction is proportional to proton velocity **u**, i.e. the collision frequency ν_{pe} is velocity-independent.

Scattering by a Cold Scatterer Species

If the scatterer species k is cold, i.e. has distribution function

$$f_k(u) = \frac{1}{4\pi\tilde{v}^2}\delta(u), \tag{A9}$$

Eqs. (A3) can be evaluated trivially. The result is

$$\mathbf{F} = -\frac{4\pi n_k e^4 Z_k^2 Z_j^2}{m_j^2 u^3}\frac{m_j + m_k}{m_k}\lambda\mathbf{u} \equiv -\nu_{jk}\mathbf{u}\,, \tag{A10a}$$

$$D_{\parallel} = 0\,, \tag{A10b}$$

$$D_{\perp} = \frac{4\pi n_k e^4 Z_k^2 Z_j^2}{m_j^2 u}\lambda \equiv \nu_{jk}\frac{m_k}{m_j + m_k}u^2. \tag{A10c}$$

For cold scatterers, the parallel diffusion vanishes, and the Fokker-Planck equation reduces to friction and transverse diffusion. In the limit $m_j/m_k \to 0$, there is no energy exchange during an elastic collision, and therefore one expects the Fokker-Planck equation for $f_j(\mathbf{v})$ to reduce to diffusion on a constant-energy spherical surface. In the more general case where the scatterers are cold, but it is not assumed that m_j/m_k is small, one expects the Fokker-Planck equation to reduce to diffusion on a constant-energy spherical surface, plus self-similar contraction of the spherical surface due to friction. We shall now prove that this is correct, by rewriting Eqs. (A1,10) in spherical coordinates (u,θ,ψ), centered on the particle velocity **u**, with θ the polar angle and ψ the azimuthal angle. Noting that $D_{\perp}$ is the only nonvanishing component of the tensor **D**, the second term on the RHS of (A1) can be written

$$\frac{1}{2}\frac{\partial^2}{\partial \mathbf{v}\partial \mathbf{v}}:\mathbf{D}f_j = -\frac{1}{u^2}\frac{\partial}{\partial u}uD_\perp f_j + \frac{1}{2u^2\sin\theta}\left(\frac{\partial^2}{\partial\theta^2}\sin\theta - \frac{\partial}{\partial\theta}\cos\theta\right)D_\perp f_j + \frac{1}{2u^2\sin^2\theta}\frac{\partial^2}{\partial\psi^2}D_\perp f_j. \tag{A11}$$

Since $D_\perp$ from Eq. (A10c) is independent of θ and φ, this reduces to

$$\frac{1}{2}\frac{\partial^2}{\partial \mathbf{v}\partial \mathbf{v}}:\mathbf{D}f_j = -\frac{1}{u^2}\frac{\partial}{\partial u}uD_\perp f_j + \frac{D_\perp}{2u^2}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\psi^2}\right)f_j. \tag{A12}$$

According to Eqs. (A10),

$$\mathbf{F} = -\frac{\mathbf{u}}{u^2}\frac{m_j+m_k}{m_k}D_\perp, \tag{A13}$$

and thus we can combine the terms of Eq. (A1) to get

$$\frac{\partial f_j}{\partial t} = -\frac{1}{u^2}\frac{\partial}{\partial u}u^2\tilde{F}f_j + \frac{D_\perp}{2u^2}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\psi^2}\right)f_j, \tag{A14}$$

where

$$\tilde{F} = -\frac{4\pi n_k e^4 Z_k^{\,2} Z_j^{\,2}}{m_j^{\,2}u^2}\frac{m_j}{m_k}\lambda \equiv -\nu_{jk}\frac{m_j}{m_j+m_k}u. \tag{A15}$$

The first term of (A14) is the friction term in spherical coordinates. Equation (A15) shows that this term vanishes in the limit of light particles scattering off heavy particles. The second term of (A14) is just diffusion on the spherical constant energy shell.

Equation (A14) has an exact solution in the form

$$f_j(u,\theta,\varphi) = \frac{1}{4\pi u^2}\delta\big(u-u_0(t)\big)g(\theta,\varphi,t), \tag{A16}$$

where $u_0(t)$ is given by

$$\frac{du_0}{dt} = \tilde{F}, \tag{A17}$$

and $g(\theta,\varphi,t)$ is the solution to

$$\frac{\partial g}{\partial t} + \frac{d\tilde{F}}{du}g = \frac{D_\perp}{2u^2}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\psi^2}\right)g. \tag{A18}$$

The second term on the LHS of (A18) is small in the limit of light particles scattering off heavy particles. If the distribution $f_j(\mathbf{v})$ initially has zero energy spread, then the Fokker-Planck equation separates into diffusion on a constant-energy shell, and self-similar collapse of the energy shell at the friction rate given by Eq. (A18). There is no spread in the particle energies at any given time. This conclusion is relevant to the evolution of the proton velocity distribution under the influence of p-^{11}B collisions, since both the protons and the boron are assumed to begin as cold beams with very little energy spread.

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19. For example, see Sec. 4 of Ref. 2, "Steady State Reactor Operation."

20. Momentum exchange collision frequencies are defined as $\nu_{jk} \equiv |V_j - V_k|^{-1}(dV_j/dt)]$, with the scatterers assumed to be in steady state. Analytic expressions are derived in the Appendix, Eqs. (A8,A10,A15). Numerical values can be found in NRL publication NRL/PU/6790—98-358, *NRL Plasma Formulary*, Revised 1998, pp. 31-35 (available from J. D. Huba, NRL Code 6794).

21. See Ref. 1, second paragraph of p. 1421: "Particles with $v_\varphi < 0$ are confined and particles with $v_\varphi > 0$ are released promptly."

22. N. Rostoker and M. W. Binderbauer, response to Ref. 13, reproduced in Ref. 14, page 21. The notation has been changed a bit for consistency with the notation used in the present paper.

23. A slightly more detailed analysis shows that if a steady state is maintained by some external force on the protons, a spread in proton energy relative to the boron also slowly develops, in addition to the transverse diffusion of the protons on the constant-energy shell, i.e. the constant-energy shell thickens.

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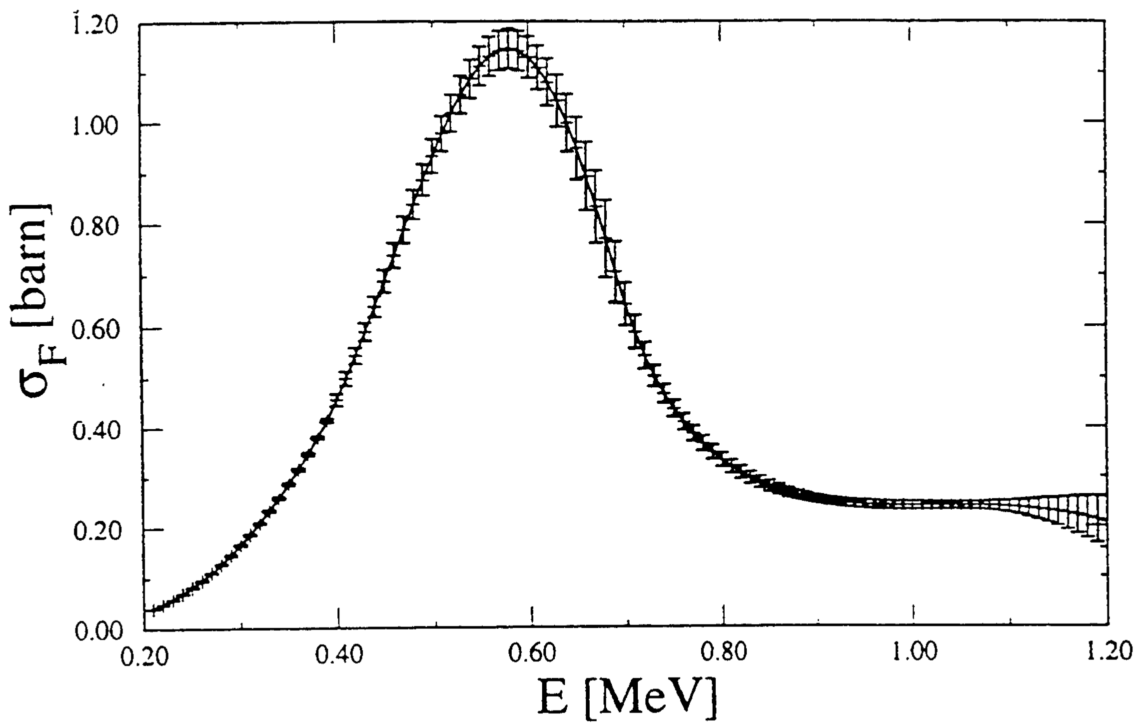

Figure 1. Fusion cross-section for unpolarized p-^{11}B, as a function of collision energy E, according to Becker et al.[17] (A smaller cross-section is given by Ref. 16.) RBM argue that the Becker cross-section can be further increased by a factor of 1.6 for 100% spin-polarized fuel.

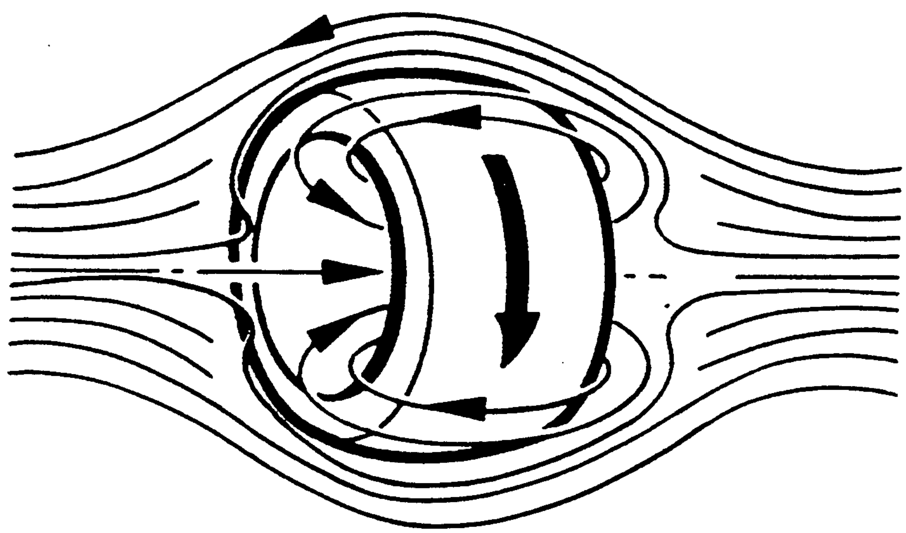

Figure 2. Geometry of the field-reversed configuration for the CBFR, showing the magnetic field lines and the rotating annular proton beam.

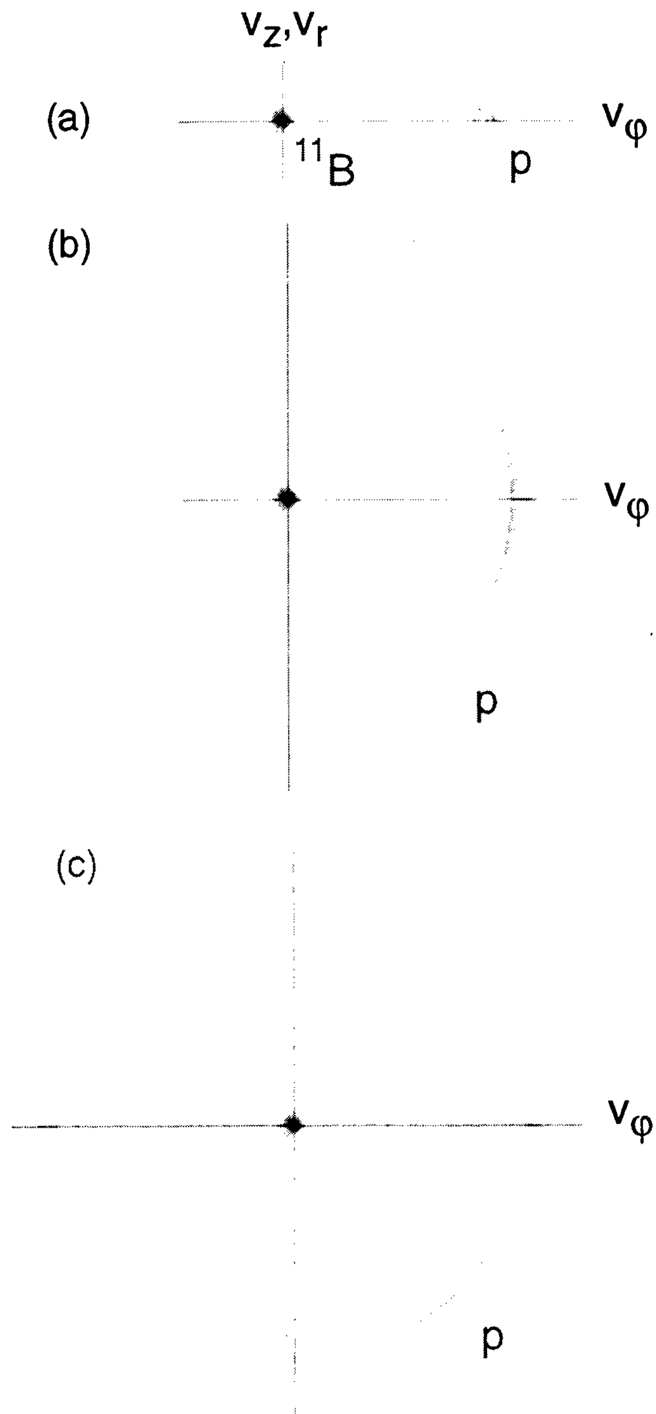

Figure 3. (a) Ion velocity distributions for the cold-colliding-beam equilibrium assumed for the CBFR. The dark circle at the origin represents the ^{11}B velocity distribution, while the circle to the right represents the proton velocity distribution.
(b) Proton velocity distribution resulting from p-^{11}B momentum scattering, at time ~ν_{pB}^{-1}. The protons have diffused over the right hemisphere of the constant-u sphere.
(c) Proton velocity distribution, at time >ν_{pB}^{-1}, if p-^{11}B momentum scattering were the only process at work. The proton diffusion has progressed to the point where the proton distribution is uniform on the constant-u sphere.

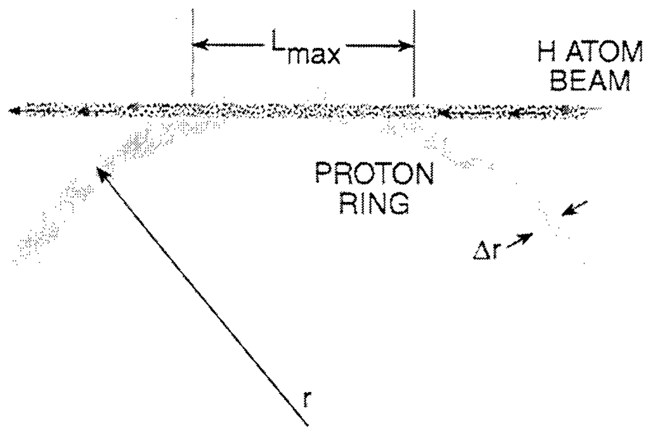

Figure 4. Geometry for neutral H atom injection into the CBFR. To be trapped, the H atoms must be ionized while traversing the chord of length $\leq L_{max}$.